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Continuum dual theory of the transition in 3D lattice superconductor

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Abstract. A recently proposed form of dual theory for three dimensional superconductor is rederived starting from the lattice electrodynamics and studied by renormalization group. The superfluid density below and close to the transition vanishes as an inverse of the correlation length for the disorder field. The corresponding universal amplitude is given by the fixed point value of the dual charge, and it is calculated to the leading order. The continuum dual theory predicts the divergence of the magnetic field penetration depth with the XY exponent, in contradiction to the results obtained from the Ginzburg–Landau theory for the superconducting order-parameter. Possible reasons for this difference are discussed.

Duality transformations in statistical mechanics have led to important insights into the nature of phase transitions in various systems, ever since the discovery of self-duality in two-dimensional (2D) Ising model by Kramers and Wannier [1]. This concept has also proved important in studies of the phase transition in 2D XY model, where dual partition function factorizes into spin-wave and vortex contributions [2], and therefore provides a natural framework for description of the Kosterlitz–Thouless transition. Similarly, the phase transition in 3D XY model can be understood within a dual approach, where the basic objects, instead of vortices, now become vortex loops. The critical temperature is identified as the point of proliferation of vortex loops through the system. The high-temperature phase of the original XY model corresponds to the low-temperature phase of the dual model, in which the disorder field that describes vortex loops has condensed [3]. The critical exponent for the correlation length calculated within the dual approach [4] is in very good agreement with the standard value, confirming the correctness of the physical picture of condensation of vortex loops at the transition in 3D XY model.

Critical behaviour of superconductors is described by the Ginzburg–Landau theory for a complex scalar field which, Cooper pairs being charged, is coupled to a fluctuating gauge potential. The nature of the phase transition in this system has long been a matter of debate: while close to four dimensions the transition is always first-order [5, 6], in three dimensions, it is believed that it can be both first and second order, depending on the value of the Ginzburg–Landau parameter κ [7–10]. While some consensus has emerged on the overall topology of the phase diagram in the 3D case, the precise nature of the continuous transition in the theory is still controversial. Recent renormalization group analysis of the Ginzburg–Landau theory for superconducting order parameter yields a novel critical behaviour, distinct from that of the neutral superfluid [9, 10]. Monte Carlo and analytical studies of the lattice version of the theory suggest, on the other hand, that the transition still belongs to the XY universality class [11, 7]. As a step towards a better understanding of these issues, in this paper the theory for 3D lattice superconductor is revisited. Under a certain plausible, but unproven assumption, the dual partition function of lattice superconductor may be expressed in a form that has a simple continuum limit. Based on different arguments, the same form of the continuum dual theory was previously proposed in [12], and studied in [13]. However, there are some important differences between the assumptions and the conclusions of the present work and those of the previous study [13]. The continuum dual model is analysed by renormalization group in 3D and the transition described by this theory is confirmed to be in the universality class of 3D XY model. The stiffness constant proportional to the superfluid density vanishes as a power-law close to the transition, with a universal amplitude given by the fixed point value of the dual charge. In combination with the result on the anomalous dimension of the gauge field at the stable fixed point in the original theory [9], the continuum dual theory suggests that the magnetic field penetration depth diverges with the power $\nu_{xy} \simeq 2/3$. This agrees with the result of Peskin [14], but not with the conclusions of [13]. It also disagrees with the exponent derived from the Ginzburg–Landau theory [9, 10]. Possible reasons for this difference are considered.

The partition function for lattice superconductor (3D lattice electrodynamics) is defined by [14, 15]:

$$Z = \int_A' \int_\theta \exp(K_0 \sum \cos(\theta_{n+\hat{\mu}} - \theta_n - A_{n\mu}) - \frac{1}{2e^2} \sum (\nabla \times A)^2_{n\mu}) \tag{1}$$

where the sums run over sites and links of a 3D quadratic lattice, $\hat{\mu} = \hat{x}, \hat{y}, \hat{z}$, $n\mu$ denotes an oriented link between n and $n + \hat{\mu}$ sites, integrals over gauge potential in the transverse gauge $\nabla \cdot A = 0$ run over whole real axis, integrals over phases are from $-\pi$ to π, and $\nabla \times A$ and $\nabla \cdot A$ are the lattice curl and the lattice divergence, respectively. e is the charge of a Cooper pair and K_0 is the stiffness constant proportional to the superfluid density in the Meissner phase. The lattice superconductor corresponds to the extreme type-II limit of the Ginzburg–Landau model where the phase transition between normal and superconducting phases is expected to be continuous [9, 10]. In the Villain approximation [2] the phases can be integrated out exactly, and the theory may be expressed in terms of integer-valued link variables $\{m_{n\mu}\}$ coupled to the gauge field:

$$Z = \int_A' \sum_m' \exp(-\frac{1}{2K_0} \sum m^2_{n\mu} + i \sum m_{n\mu} A_{n\mu} - \frac{1}{2e^2} \sum (\nabla \times A)^2_{n\mu}) \tag{2}$$

where primes on the integral and on the sum, as throughout the paper, denote the conditions $\nabla \cdot A = \nabla \cdot m = 0$. The second condition forces the integer variables $\{m_{n\mu}\}$ to form closed loops. For $e = 0$ the gauge field decouples, and one is left with the standard loop representation of the XY model, with the continuous phase transition at some $K_0 = K_c$. If $K_0 = \infty$, the sum over integers $\{m_{n\mu}\}$ can be performed, and the gauge-field becomes constrained to integer values. Remarkably, in this limit the lattice superconductor again maps onto the XY model [14]. Building on this observation, it has been argued that even for finite K_0 the lattice superconductor should exhibit the *inverted* XY transition [11]. Here we note that the partition function can be rewritten as

$$Z = \int_A' \int_h' \sum_m \exp(-\frac{1}{2K_0} \sum (\nabla \times h)^2_{n\mu} + i \sum h_{n\mu} (\nabla \times A)_{n\mu} - \frac{1}{2e^2} \sum (\nabla \times A)^2_{n\mu} + i2\pi \sum m_{n\mu} h_{n\mu}) \tag{3}$$

so that the gauge field A can be easily integrated out. Since the field h is purely transverse, integer variables $\{m_{n\mu}\}$ can be taken to be transverse as well, and

$$Z = \lim_{t\to 0} \int_h' \sum_m' \exp(i2\pi \sum m_{n\mu} h_{n\mu} - \frac{1}{2K_0} \sum (\nabla \times h)^2_{n\mu} - \frac{e^2}{2} \sum h^2_{n\mu} - \frac{t}{2} \sum m^2_{n\mu}) \tag{4}$$

where we added a chemical potential term. Up to Villain approximation, this is the same as

$$Z = \lim_{t\to 0} \int_h' \int_\theta \exp(\frac{1}{t} \sum \cos(\theta_{n\mu} - \theta_n - 2\pi h_{n\mu}) - \frac{1}{2K_0} \sum (\nabla \times h)^2_{n\mu} - \frac{e^2}{2} \sum h^2_{n\mu}). \tag{5}$$

When $e = 0$, the dual partition function (5) is identical to the $K_0 \to \infty$ limit of theory in (1). In this limit therefore, the transition is at $e^2 = e_c^2 = 4\pi^2 K_c$, as originally found by Peskin [14]. Apart from the limit $t \to 0$, the last expression is completely analogous to the partition function for the lattice superconductor, equation (1), except that the dual gauge-field h is *massive*. Finite t adds a small chemical potential for the loop variables $\{m_{n\mu}\}$ in equation (4). This suggests that the limit $t \to 0$ is regular, in the sense that the nature of the phase transition is not changed by leaving t finite. This is the same assumption as the one made by the authors of [11] in their argument for the inverted XY transition in lattice superconductor. Unfortunately, we have been unable to check directly the validity of this assumption. Rather, we will adopt it for a moment, and return to this question after we derive its consequences. For $t \neq 0$, the dual model of the lattice superconductor should be in the same universality class as the continuum theory for a complex scalar field representing vortex loops (the disorder field) minimally coupled to the massive dual gauge-field $\vec{h}$ (with $\nabla \cdot \vec{h} = 0$) [12, 13]:

$$L = \int d^3\vec{r}[|(\nabla - i\frac{2\pi}{e}\vec{h})\Psi(\vec{r})|^2 + \mu^2|\Psi(\vec{r})|^2 + \frac{b_0}{2}|\Psi(\vec{r})|^4 + \frac{1}{2}\vec{h}^2 + \frac{1}{2\mu_h^2}(\nabla \times \vec{h})^2]. \tag{6}$$

$\mu^2 \sim (T_{c0} - T)$, T_{c0} is the mean-field transition temperature, the quartic term describes the short-range repulsion between the vortex loops, and $\mu_h^2 = e^2 K_0$.

Having a continuum version of the dual theory, the critical behaviour can be studied by standard renormalization group methods (RG). We adopt the multiplicative field-theoretic renormalization and calculate the RG factors perturbatively and directly in 3D. The perturbation theory is used only to make the reasoning more explicit, and is not essential for the main conclusions. The renormalized dual theory is:

$$L_r = \int d^3\vec{r}[Z_\Psi|(\nabla - i\frac{2\pi}{e}\vec{h})\Psi|^2 + m'^2|\Psi|^2 + \frac{b'}{2}|\Psi|^4 + \frac{1}{2}\vec{h}^2 + \frac{Z_h}{2\mu_h^2}(\nabla \times \vec{h})^2]. \tag{7}$$

Notice that the form of minimal coupling between Ψ and $\vec{h}$ is preserved and the same Ward identities hold as if $\vec{h}$ was massless. This further implies that the polarization of the field $\vec{h}$ is transverse and $\vec{h}^2$ term does not get renormalized. To the leading order in two dimensionless coupling constants $\hat{b}_0 = b_0/m$ and $\hat{g}_0 = (2\pi/e)^2(\mu_h^2/m)$ we obtain:

$$Z_h = 1 + \frac{\hat{g}_0}{24\pi} \tag{8}$$

$$Z_\Psi = 1 - \frac{2\hat{g}_0}{3\pi(1+(\mu_h/m))} \tag{9}$$

$$b' = b_0 - \frac{5}{8\pi}\frac{b_0^2}{m} - \frac{1}{2\pi}\frac{(2\pi\mu_h/e)^4}{\mu_h}. \tag{10}$$

With the renormalized coupling constants defined as $\hat{g} = (2\pi/e)^2(m_h^2/m)$, $m_h^2 = \mu_h^2/Z_h$, $m = m'/Z_\Psi^{1/2}$ and $\hat{b} = \hat{b}'/Z_\Psi^2$, the one-loop β-functions are:

$$\beta_g \equiv \frac{d\hat{g}}{d\log m} = -\hat{g} + \frac{1}{24\pi}\hat{g}^2 \tag{11}$$

$$\beta_b \equiv \frac{d\hat{b}}{d\log m} = -\hat{b} + \frac{5}{8\pi}\hat{b}^2 - \frac{2}{3\pi}\frac{\hat{b}\hat{g}(2+m_h/m)}{(1+(m_h/m))^2} + \frac{1}{2\pi}\hat{g}^2\frac{m}{m_h}. \tag{12}$$

First, there are two unstable fixed points with vanishing dual charge $\hat{g}_c = 0$: $\hat{b}_c = 0$ and $\hat{b}_c = 8\pi/5$. Note that the last two terms in β_b depend not only on $\hat{b}$ and $\hat{g}$, but on the third dimensionless combination of the coupling constants m/m_h as well. The phase transition in the theory is achieved by tuning the temperature, that is by letting $m \to 0$, with the scaling of m_h determined by β_g. Close to the attractive fixed point with a finite dual charge $\hat{g} = \hat{g}_c \neq 0$:

$$(\hat{g} - \hat{g}_c) = \text{const} \cdot m^{\beta_g'(\hat{g}_c)} \tag{13}$$

so that,

$$\frac{(2\pi)^2}{e^2}m_h^2 = \text{const} \cdot m^{1+\beta_g'(\hat{g}_c)} + \hat{g}_c m. \tag{14}$$

Since $\beta_g'(\hat{g}_c) = 1 > 0$, when $m \to 0$ the renormalized stiffness constant $K = m_h^2/e^2$ close to the transition approaches zero as

$$K = \frac{\hat{g}_c}{(2\pi)^2}m. \tag{15}$$

Recalling that $m \equiv \xi^{-1}$, where ξ is the correlation length of the disorder field, we see that the result is in accordance with the Josephson relation in 3D [16]. To the lowest order in dual charge, the universal amplitude is

$$\lim_{m\to 0}\frac{K}{m} = \frac{6}{\pi}. \tag{16}$$

Now it becomes obvious that the last two terms in β_b vanish as the critical temperature is approached because $m_h/m \to \infty$, in spite of the scaling of dual charge towards a finite fixed point value. Consequently, the attractive fixed point with $\hat{g}_c \neq 0$ is located at the same $\hat{b}_c = 8\pi/5$ as the pure XY fixed point. As may have been expected, the fluctuations of the massive dual gauge-field do not influence the critical behaviour of the theory, which is in the universality class of 3D XY model [12, 13]. This is in sharp contrast to the effects of the massless gauge-potential in the Ginzburg–Landau theory [5, 9, 10]. We proved this statement here only to the lowest order, but clearly it must persist to all orders in perturbation theory. One could integrate out the dual gauge field $\vec{h}$ from the beginning and the only effect of that (apart from introducing some irrelevant terms) would be the modification of the bare quartic term for the disorder field.

From the partition function for the lattice superconductor, equation (1), it follows that the magnetic field penetration depth λ scales with the stiffness constant as :

$$\lambda^{-(2-\eta_A)} \propto K \tag{17}$$

where η_A is the anomalous dimension acquired by the gauge-field at the stable fixed point in the original theory. At the neutral, unstable fixed point with $e = 0$, $\eta_A = 0$. At the charged, attractive fixed point in the theory, however, $\eta_A = 1$ in 3D [9]. This may also be understood by recalling that in the continuum limit the dimension of the charge $[e^2] = L^{-1}$, so that close to the stable fixed point scaling of the $(\nabla \times A)^2$ term with length is changed by one power. From the relations (15) and (17) it follows that

$$\lambda \propto \xi \tag{18}$$

and the exponents for the correlation length of the disorder field and for the magnetic field penetration depth are the same. This result is independent of the perturbation theory. It was only necessary that $\beta_g'(\hat{g}_c) > 0$, which is always true at a non-trivial zero of β_g. The continuum dual theory (6) thus implies that the exponent for the magnetic field penetration depth has the XY value $\nu_{xy} \simeq 0.67$.

The same form of the continuum dual theory (6) has previously been considered in [13]. However, by making some phenomenological assumptions for the coupling constants in the theory, these authors obtained an incorrect value for the exponent for the penetration depth. The essential difference with the present work is that the bare mass of the dual gauge field $\vec{h}$ was assumed to independently vanish close to the critical point as $\mu_h^2 \propto (T_c - T) \propto m^x$, $x = 1/\nu_{xy}$. The lowest order β-function for the dual charge is then modified to read:

$$\beta_g = (x-1)(\hat{g} - \frac{1}{24\pi}\hat{g}^2) \tag{19}$$

and the neutral ($\hat{g}_c = 0$) fixed point of the dual theory becomes infrared stable for any $x > 1$. Since $\beta_g'(0) = x - 1$, equation (13) then implies that

$$m_h^2 \sim m^x. \tag{20}$$

The power x does not get changed by the fluctuations of the disorder field if its assumed value is larger than unity. The authors of [13] interpreted this observation as that the exponent for the penetration depth has the mean-field value $1/2$. It is obvious, however, that any other assumption for the power x would, as long as $x > 1$, lead to a correspondingly different exponent for the penetration depth. Furthermore, the interpretation of the field theory (6) as a continuum limit of the dual lattice superconductor makes it clear that nothing should be assumed for the mass μ_h. Close to the transition, the renormalized mass m_h is then *driven* to zero by fluctuations of the disorder field, as a power law that follows from the theory.

The behaviour of the penetration depth derived here on the basis of the continuum dual theory is in disagreement with of results obtained from the Ginzburg–Landau theory for the superconducting order-parameter. Perturbative [9] and non-perturbative [10] renormalization group calculations in fixed dimension indicate that the penetration depth in the Ginzburg–Landau model diverges with the exponent $0.50 < \nu < 0.62$. The main point is that the exponent obtained from Ginzburg–Landau theory is different (and most likely smaller) than ν_{xy}. In fact, this is what one would naively expect, since the neutral, XY fixed point in the Ginzburg–Landau theory is unstable with respect to the charge, and small perturbations in that direction are attracted by the stable, charged fixed point with different exponents. One conceivable reason for this disagreement is the approximate nature of the calculations performed in [9] and [10]. One should note however, that the non-perturbative method employed in [10] leads to very accurate values for the exponents in several other problems, where there is more information available. A more interesting possibility is that the condition $t \to 0$ which we were forced to relax in order to arrive at the continuum theory, might represent a singular limit in the theory [17]. In equation (4), for $t = 0$, the summation over integers $\{m_{n\mu}\}$ would force the dual gauge field h to take strictly integer values. Finite t relaxes this constraint, and h becomes distributed around integers with a finite width of distribution proportional to t. When viewed this way, it becomes less clear that finite and zero t necessarily lead to the same physics. More detailed numerical studies should be able to shed some light on the problem.

The divergence of the penetration depth close to the superconducting transition is experimentally measurable, at least in principle. In practice, unfortunately, this is made prohibitively difficult by smallness of the critical region for the gauge-field fluctuations, which is rather narrow (even in the high-temperature superconductors) due to small value of the effective charge (Ginzburg–Landau parameter $\kappa >> 1$). The presently accessible critical region corresponds to the vicinity of the unstable neutral fixed point in the Ginzburg–Landau theory, at which the penetration depth diverges with the exponent $\nu_{xy}/2$ (because $\eta_A = 0$ at this fixed point) [18]. We should note here, however, that there are other physical systems where the critical region for the gauge-field fluctuations is much wider [19, 20] and which offer more hope for the experimental resolution of the question of the exponents at the charged critical point.

In summary, the message of this paper is twofold. First, we studied the phase transition in 3D lattice superconductor within a continuum version of the dual theory which yields the transition to be in the XY universality class. The superfluid density vanishes as inverse of the correlation length of the disorder field. In contrast to the previous claims, dual charge is a relevant coupling whose fixed point value determines the universal amplitude for the scaling of the stiffness constant. This amplitude is calculated to the lowest order in perturbation theory. Finally, we have shown that within the continuum dual theory the magnetic field penetration depth diverges with the exponent $\nu_{xy} \simeq 2/3$. Second, it is pointed out that the critical behaviour obtained on the basis of the continuum dual theory is in disagreement with the recent results from Ginzburg–Landau theory for the superconducting order-parameter. Possible reasons for this disagreement are discussed and experimental consequences are mentioned.

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